

**NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS**

REPORT No. 896

**PRESSURE-SENSITIVE SYSTEM FOR
GAS-TEMPERATURE CONTROL**

By RICHARD S. CESARO and NORMAN MATZ



1948

AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbrevia- tion	Unit	Abbrevia- tion
Length.....	l	meter.....	m	foot (or mile).....	ft (or mi)
Time.....	t	second.....	s	second (or hour).....	sec (or hr)
Force.....	F	weight of 1 kilogram.....	kg	weight of 1 pound.....	lb
Power.....	P	horsepower (metric).....		horsepower.....	hp
Speed.....	V	kilometers per hour.....	kph	miles per hour.....	mph
		meters per second.....	mps	feet per second.....	fps

2. GENERAL SYMBOLS

W	Weight= mg	ν	Kinematic viscosity
g	Standard acceleration of gravity= 9.80665 m/s^2 or 32.1740 ft/sec^2	ρ	Density (mass per unit volume)
m	Mass= $\frac{W}{g}$		Standard density of dry air, $0.12497 \text{ kg-m}^{-3}$ at 15°C and 760 mm ; or $0.002378 \text{ lb-ft}^{-3} \text{ sec}^2$
I	Moment of inertia= mk^2 . (Indicate axis of radius of gyration k by proper subscript.)		Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb/cu ft
μ	Coefficient of viscosity		

3. AERODYNAMIC SYMBOLS

S	Area	i_w	Angle of setting of wings (relative to thrust line)
S_w	Area of wing	i_t	Angle of stabilizer setting (relative to thrust line)
G	Gap	Q	Resultant moment
b	Span	Ω	Resultant angular velocity
c	Chord	R	Reynolds number, $\rho \frac{Vl}{\mu}$ where l is a linear dimen- sion (e.g., for an airfoil of 1.0 ft chord, 100 mph , standard pressure at 15°C , the corresponding Reynolds number is $935,400$; or for an airfoil of 1.0 m chord, 100 mps , the corresponding Reynolds number is $6,865,000$)
A	Aspect ratio, $\frac{b^2}{S}$	α	Angle of attack
V	True air speed	ϵ	Angle of downwash
q	Dynamic pressure, $\frac{1}{2}\rho V^2$	α_0	Angle of attack, infinite aspect ratio
L	Lift, absolute coefficient $C_L = \frac{L}{qS}$	α_i	Angle of attack, induced
D	Drag, absolute coefficient $C_D = \frac{D}{qS}$	α_a	Angle of attack, absolute (measured from zero- lift position)
D_0	Profile drag, absolute coefficient $C_{D_0} = \frac{D_0}{qS}$	γ	Flight-path angle
D_i	Induced drag, absolute coefficient $C_{D_i} = \frac{D_i}{qS}$		
D_p	Parasite drag, absolute coefficient $C_{D_p} = \frac{D_p}{qS}$		
C	Cross-wind force, absolute coefficient $C_c = \frac{C}{qS}$		

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**Flight Propulsion Research Laboratory
Cleveland, Ohio**

National Advisory Committee for Aeronautics

Headquarters, 1724 F Street NW, Washington 25, D. C.

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SUMMARY

A thermodynamic relation is derived and simplified for use as a temperature-limiting control equation involving measurement of gas temperature before combustion and gas pressures before and after combustion. For critical flow in the turbine nozzles of gas-turbine engines, the control equation is further simplified to require only measurements upstream of the burner. Hypothetical control systems are discussed to illustrate application of the control equations.

INTRODUCTION

The problem of gas-temperature control is becoming increasingly important for high-temperature applications, such as the gas-turbine power plant for aircraft. The acceleration of conventional gas-turbine power plants is currently induced by increasing the fuel flow to the engine. Because fuel flow is increased before the air flow increases, such a process of acceleration is accompanied by an increase in combustion-gas temperature to values that may exceed the temperature limitation of the engine. Temperature-limiting control is therefore required in order to eliminate the possibility of engine failure from overheating during this transient phase of operation.

The temperature limitation of an engine is determined by the temperature at which any engine component will fail. Because of the poor response characteristics inherent in a system of temperature control dependent on the sensing of metal temperatures, such a method is considered impracticable. The sensing and the control of combustion-gas temperatures appear to be more advantageous than an attempt to control directly the temperature of the critical engine component.

Limiting control of gas temperatures presents a difficult problem in that direct means of sensing combustion-gas temperatures of the magnitude encountered in gas-turbine engines are as yet unsatisfactory. The usual means of sensing temperature, such as thermocouples and resistance wires, are subject to errors resulting from radiation, conduction, and oxidation. In addition to these errors, the life of such instruments is comparatively short at the high temperatures to which they would be subjected. The thermodynamic methods presented herein were conceived to circumvent such difficulties.

Any means of sensing temperature that is selected for control purposes should be accurate, respond quickly, and pro-

vide a response that is easily incorporated into a control system. Measurement and interpretation of the thermodynamic changes that occur in the working fluid during the combustion process offer possibilities for meeting these requirements and obtaining temperature indications that can be used for control application. Temperature changes of the combustion gases are accompanied by pressure changes that are in effect instantaneous, are unaffected by errors of radiation and conduction, and can be utilized in a control system without modification or amplification.

Investigations have been made correlating temperature rise across the combustion chamber with pressure changes. These correlations, however, depend on combustion-chamber pressure losses requiring a calibration of the particular combustion chamber and the method appears impractical for control purposes.

A theoretical equation based on the thermodynamic properties of the combustion gas that correlates gas-temperature ratio with pressures upstream and downstream of the combustion zone was developed at the NACA Cleveland laboratory during 1947 and is presented herein. Means of simplifying this equation for control purposes are discussed and hypothetical control systems are presented to illustrate application of the control equations.

DERIVATION OF CONTROL EQUATIONS

GENERAL CONTROL EQUATION

Determination of the final temperature of the gas from the thermodynamic relations involved is possible without direct sensing of the temperature in the hot gas stream by equating the mass flow of gas in a duct upstream of the point of addition of heat (station 1) to the mass flow downstream of the point of addition of heat (station 2). (The symbols used herein are defined in appendix A.)

The flow at station 2 equals the air flow at station 1 plus the fuel flow f , that is,

$$a_2 = a_1 + f$$
$$a_2 = \left(1 + \frac{f}{a_1}\right) a_1$$

When the equivalent gas-flow equations are substituted for a_1 and a_2 ,

$$E_2 A_2 \phi_2 \sqrt{2g\rho_2 \Delta P_2} = \left(1 + \frac{f}{a_1}\right) E_1 A_1 \phi_1 \sqrt{2g\rho_1 \Delta P_1} \quad (1)$$

The density ρ may be replaced by its equivalent $\frac{p}{RT}$ and ϕ by its corresponding relation ϕ' , as shown in appendix B:

$$E_2 A_2 \phi'_2 \sqrt{\frac{p_2}{T_2} \Delta P_2} = \left(1 + \frac{f}{a_1}\right) E_1 A_1 \phi'_1 \sqrt{\frac{p_1}{T_1} \Delta P_1} \quad (2)$$

The unknown variable, total temperature T_2 , may now be obtained from the equation

$$T_2 = \left[\frac{E_2 A_2 \phi'_2}{E_1 A_1 \phi'_1} \frac{1}{\left(1 + \frac{f}{a_1}\right)} \right]^2 \frac{p_2 \Delta P_2}{p_1 \Delta P_1} T_1 \quad (3)$$

Equation (3) offers a means of determining total temperature T_2 and may be simplified for control application by using the following assumptions:

1. The area ratio A_2/A_1 is a known constant and may be obtained by direct measurement; or by use of equation (3), it may be found by calibrating the system without heating the gas.

2. In a particular design, the operating range of temperatures and of pressure ratios is comparatively small and the conversion factor ϕ may be selected as constant. The factor ϕ is a multiplication factor by which the hydraulic equation is converted to the compressible-flow equation. The equation for the conversion factor ϕ therefore varies with each combination of static pressure p or total pressure P and static temperature t or total temperature T used in the hydraulic equation so that the compressible-flow equation is always obtained. The expression for determining the actual value of the conversion factor ϕ' when $\rho = \frac{p}{RT}$ is developed in appendix B for the case in which the density ρ is proportional to the static pressure p divided by the total temperature T . This density relation is desirable for this analysis because it incorporates the total rather than the static temperature and also because the numerical value of the conversion factor ϕ' for each pressure ratio and value of the ratio of specific heats γ , where γ is a function of tempera-

3000° R, the conversion factor ϕ' is approximately 0.945. The curve for $\gamma=1.4$, which represents an air temperature of about 500° R, is also shown.

3. The area multiplier for thermal expansion E_1 has a negligible change because relatively little change in temperature occurs at station 1 over the range of engine operating conditions.

4. The area multiplier for thermal expansion E_2 increases to a value slightly above 1 with an increase in the fuel-air ratio f/a_1 and the ratio $\frac{E_2}{1 + \frac{f}{a_1}}$ remains a constant. For ex-

ample, with an assumed burner efficiency of 50 percent, the value of $\frac{E_2}{1 + \frac{f}{a_1}}$ decreases from 0.995 to 0.990 with a temper-

ature differential increasing from 400° to 1600° F between stations 1 and 2.

If the factors A_2/A_1 , ϕ'_2/ϕ'_1 , E_1 , and $\frac{E_2}{1 + \frac{f}{a_1}}$ are combined into a single constant K , equation (3) becomes

$$T_2 = K^2 \frac{p_2 \Delta P_2}{p_1 \Delta P_1} T_1 \quad (4)$$

This equation may be adapted for control application. The average values of velocity pressures ΔP_1 and ΔP_2 may be obtained by using single pitot-static tubes at each station in a gas stream of known velocity distribution.

Equation (4) may be further simplified for the case in which the ratio p_2/p_1 may be considered constant. Although an investigation should be made into the feasibility of making this assumption for any particular application, it will serve to simplify equation (4) for illustrating a unique hydraulic control system subsequently explained. For this condition, combination of the constants K^2 and p_2/p_1 into a single constant K' and substitution into equation (4) gives

$$T_2 = K' \frac{\Delta P_2}{\Delta P_1} T_1 \quad (5)$$

CONTROL EQUATION FOR CONDITION OF CRITICAL FLOW

Another simplification of temperature equation (3) may be made when critical flow exists at station 2 after the gas is heated. Equation (3) may be rewritten as

$$T_2 = \left[\frac{E_2 A_2}{E_1 A_1} \frac{1}{\phi'_1 \left(1 + \frac{f}{a_1}\right)} \right]^2 (\phi'_2)^2 \frac{p_2 \Delta P_2}{p_1 \Delta P_1} T_1 \quad (6)$$

where $P_2 - p_2$ replaces its equivalent ΔP_2 , and ϕ'_2 is factored out of the bracketed term.

For the condition of critical flow, the pressure ratio p_2/P_2 is

$$\frac{p_2}{P_2} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \quad (7)$$

This equation may be substituted in equation (B8) (appendix B) for its equivalent r and an expression for $(\phi'_2)^2$ in terms of γ is obtained

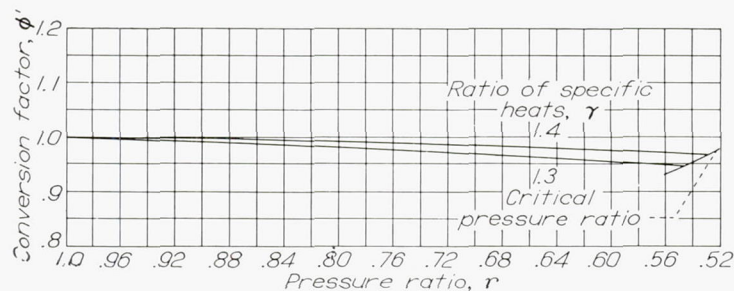


FIGURE 1.—Conversion factor of hydraulic equation to compressible air-flow equation.

$$\phi' = \sqrt{\frac{1}{r^\gamma} \left(\frac{1-\gamma}{r^\gamma - 1} \right)^\gamma}; r = \frac{\text{static pressure}}{\text{total pressure}}$$

ture, is more nearly equal to 1 than for any other relation of the density ρ . The deviation of the conversion factor ϕ' from 1 is greatest at critical flow (fig. 1) and increases with increasing temperature because of the change in the ratio of specific heats γ . At the point of critical flow and a value of γ of 1.3, which corresponds to an air temperature of about

$$(\phi'_2)^2 = \frac{\frac{\gamma}{2} \left(\frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}}}{1 - \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}} \quad (8)$$

The terms in the factor $\left[\frac{E_2 A_2}{E_1 A_1} \frac{1}{\phi'_1 \left(1 + \frac{f}{a_1} \right)} \right]$ in equation (6)

are assumed constant in the manner previously discussed for determining the constant K in equation (4) and are designated by the constant B . Substituting B and the equivalent of p_2 from equation (7) and $(\phi'_2)^2$ from equation (8) causes equation (6) to become

$$T_2 = B^2 \frac{\frac{\gamma}{2} \left(\frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}}}{1 - \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}} \frac{P_2 \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}}{p_1} \frac{P_2 - P_2 \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}}{\Delta P_1} T_1$$

This equation simplifies to

$$T_2 = B^2 \frac{\gamma}{2} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \frac{P_2^2}{p_1 \Delta P_1} T_1 \quad (9)$$

The factor $\frac{\gamma}{2} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}$ varies about 5 percent with a change in temperature from 500° to 3000° R. In the case of a temperature-limiting control for which the value of total temperature T_2 is predetermined, the value of this factor may also be predetermined. A constant B' may then be used to replace the term $\frac{T_2}{\frac{\gamma}{2} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}}$ for a particular value of T_2 and equation (9) may be expressed as

$$B' = \frac{P_2^2 T_1}{p_1 \Delta P_1} \quad (10)$$

In the operation of current gas-turbine engines in which the ratio P_2/P_1 remains substantially constant, equation (10) may be written as

$$B'' = \frac{P_1^2 T_1}{p_1 \Delta P_1} \quad (11)$$

where B'' is a constant. This control equation is unusual because sensing elements are required only before the combustion zone.

The application of a control operating in accordance with equation (10) or (11) can be illustrated by reference to the steady-state operating curve of a typical turbojet engine (fig. 2). The control operating line is shown superimposed on this engine-operating curve. In this engine, critical flow exists in the turbine nozzle box for engine speeds above 8000 rpm. For speeds less than this value, the nozzle-box Mach number M is less than 1. The maximum operating

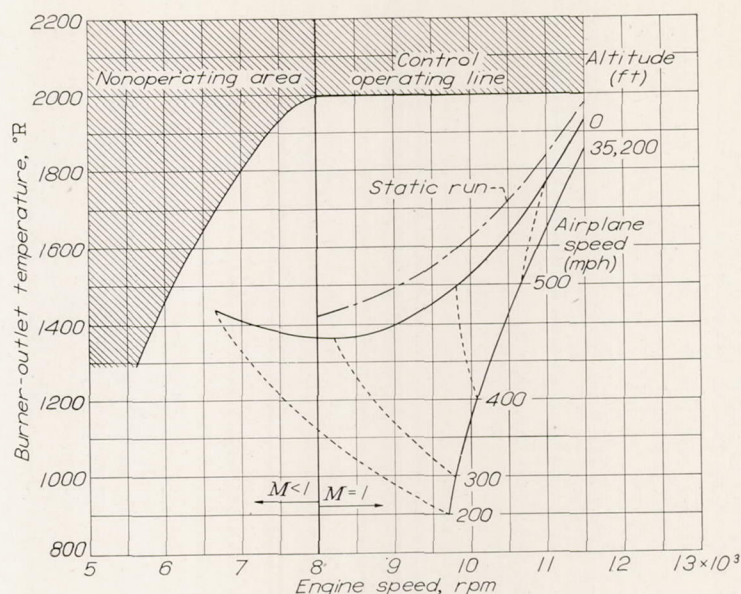


FIGURE 2.—Performance of current turbojet engine showing temperature-limit operating curve. Engine idling speed, approximately 4000 rpm; landing specifications require minimum speed of 8000 rpm. Nozzle-box Mach number, M .

temperature shown, 2000° R, is the maximum permissible gas temperature specified for this particular engine. Below an engine speed of 8000 rpm, the temperature allowed by the control decreases with decreasing engine speeds; the values shown in figure 2 were determined from equation (10) and from typical engine data. The difference between the steady-state temperature at any engine speed and altitude and the corresponding temperature allowed by the control is the temperature differential available for acceleration. The maximum permissible gas temperature is allowed by the control only for those conditions of engine operation for which critical flow exists. Most normal operation occurs at engine speeds above 8000 rpm where the control allows operation at the maximum permissible temperature. From the shape of the operating curve, the temperatures required for steady-state operation at low engine speeds apparently would not be allowed by the control. Making the control inoperative below some speed that must be determined from engine data therefore becomes necessary.

APPLICATION OF CONTROL EQUATIONS

A system designed to control total temperature T_2 by action based on the values of the other variables of equation (5) is schematically shown in figure 3. Equation (5) was selected for illustration because a simpler presentation can be made than with the more basic and more generally applicable equation (4). This control system is unusual in that the required multiplication and division are accomplished by hydraulic means. Temperature control is maintained by the automatic regulation of resistance valve A in the fuel line, which regulates the fuel flow to the nozzles to limit the gas temperature to the predetermined maximum value. Valves B, C, and D can be located in any convenient fluid-flow line through which continuous flow is maintained, such as a fuel-bypass line or a lubricating-oil line. The following relation is obtained by equating the mass rate of flow through valves C and D:

$$C_C A_C \sqrt{\Delta p_C} = C_D A_D \sqrt{\Delta p_D}$$

or

$$C_C^2 A_C^2 = C_D^2 A_D^2 \frac{\Delta p_D}{\Delta p_C} \quad (12)$$

The temperature at station 1 so controls the valve area A_D that the following relation is maintained:

$$A_D^2 = G T_1$$

where G is a constant.

The velocity pressure ΔP_1 is applied to diaphragm-operated valve B, which maintains the pressure drop across valve C, Δp_C , equal to ΔP_1 . The velocity pressure ΔP_2 is applied across diaphragm-operated valve A, which controls the fuel flow to the nozzles and thereby controls the magnitude of ΔP_2 . Valve A is in balance when the pressure drop across valve D, Δp_D , equals ΔP_2 . Substitutions may therefore be made in equation (12) for A_D^2 , Δp_D , and Δp_C :

$$A_C^2 = G \frac{\Delta P_2}{\Delta P_1} T_1 \left(\frac{C_D}{C_C} \right)^2 \quad (13)$$

When equation (13) is compared with equation (5), total temperature T_2 is found to equal $\frac{K_1}{G} \left(\frac{C_C}{C_D} \right)^2 A_C^2$. Area A_C

may therefore be set to give a predetermined value of total temperature T_2 . If the actual value of total temperature T_2 is less than this value, the value of velocity pressure ΔP_2 is less than static-pressure difference Δp_D and valve A tends to open, allowing more fuel to enter the nozzles. If the actual value of total temperature T_2 is greater than this predetermined value, the opposite action occurs. If there are no other controls in the fuel-supply line to the nozzles, the control tends to maintain the total temperature T_2 as determined by area A_C . If, however, controls exist in the system that restrict fuel flow in accordance with the demands of requirements other than maintenance of a maximum gas temperature, this control will act only to limit the maximum value of total temperature T_2 to the value determined by the setting of area A_C .

A modification of this control is shown in figure 4. The operation of this second control is essentially the same as that of the first (fig. 3) except that a motion that is linear with changes in gas temperature is obtained by the addition of a servosystem. This temperature-dependent motion may be utilized in a control system that uses temperature as a control parameter in conjunction with other engine parameters for power-plant operation. Equations (11) and (12) are still valid, the only difference between the systems being in the method of control. The static-pressure drop across valve C, Δp_C , is controlled by a diaphragm-operated servovalve and a servopiston to maintain static-pressure drop Δp_C equal to the pressure difference ΔP_1 . Valve B is a diaphragm-operated throttling valve, which acts to maintain Δp_D equal to ΔP_2 . The manual lever controls a servovalve and a servopiston-regulating valve A to maintain a set total temperature T_2 . Because area A_C is a measure of total temperature T_2 , the position of the stem of valve C is a measure of temperature as indicated by the temperature indicator. The manual lever may be so set that the servo-

valve and the piston-positioning valve A determine the equilibrium position of the stem of valve C, which, in effect, determines the total temperature T_2 .

Valves B, C, and D (fig. 4) in the bypass line are continuously functioning and the position of valve C is a con-

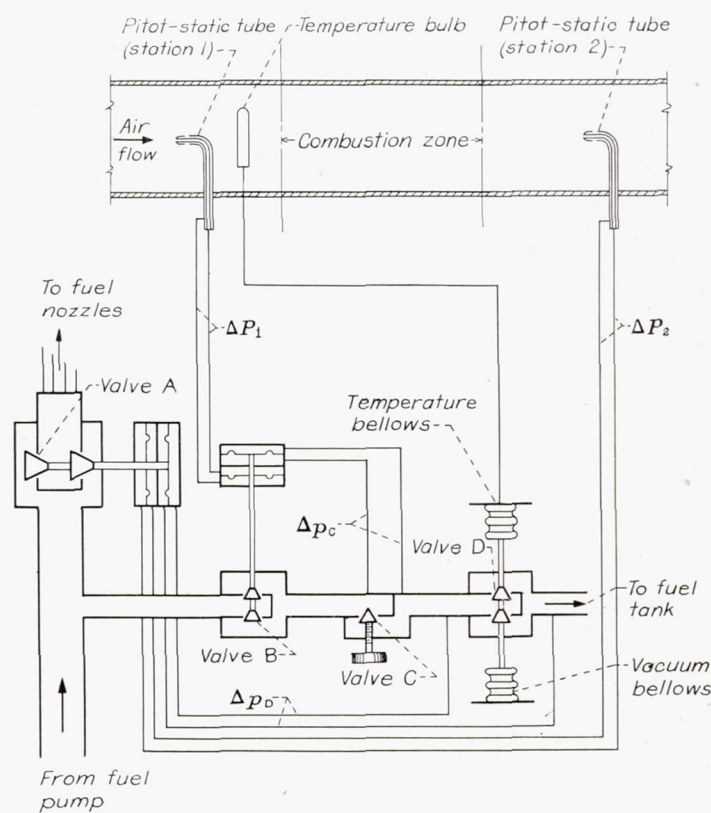


FIGURE 3.—Schematic diagram of basic temperature control.

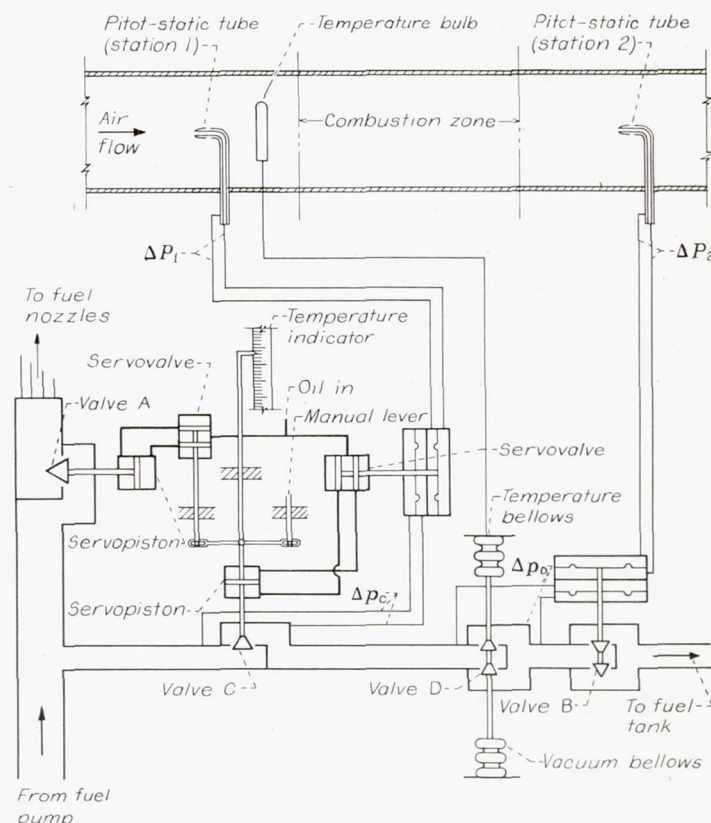


FIGURE 4.—Schematic diagram of temperature control with automatic temperature indication.

tinuous indication of temperature. The temperature indicator therefore operates when the control is used either as a temperature-limiting control or as a regulating control. This control system differs from the system shown in figure 3 in that the valve area A_C (fig. 3) is manually set by the pilot or may be preset to maintain a particular limiting temperature. This area setting indicates temperature only when the control is used as a regulating control; otherwise, the area setting is that of the limiting temperature.

A control operating according to the control equations based on critical flow at one station is schematically shown in figure 5. Either equation (10) or (11) may be used as the temperature-control equation, depending on whether total

pressure P_2 or P_1 is to be used to actuate the control. The schematic diagram in figure 5 is based on equation (11). This control is similar to those diagrammed in figures 3 and 4 in that valve A controls the fuel flow to the engine, and that valves B, C, and D are located in a convenient fuel-flow line. The density term p_1/T_1 in equation (11) is so applied to a density-compensating system that valve B controls the pressure drop Δp_C proportionally to T_1/p_1 . The area A_C is made proportional to the total pressure P_1 and the area A_D is made proportional to the constant B_2 of equation (11), which depends on the value of total temperature T_2 selected. The static-pressure difference Δp_D then so controls valve A that ΔP_1 equals Δp_D . When these substitutions are made, control equation (12) is seen to operate according to temperature equation (11).

CONCLUSIONS

The analysis indicates that determination of combustion-gas temperatures from the thermodynamic relations involving gas temperature before combustion and gas pressures before and after combustion appears practical for high-temperature application, such as gas-turbine temperature control. For critical flow in the turbine nozzles, the analysis indicates that combustion-gas temperature can be determined and controlled from measurements taken only upstream of the combustion zone.

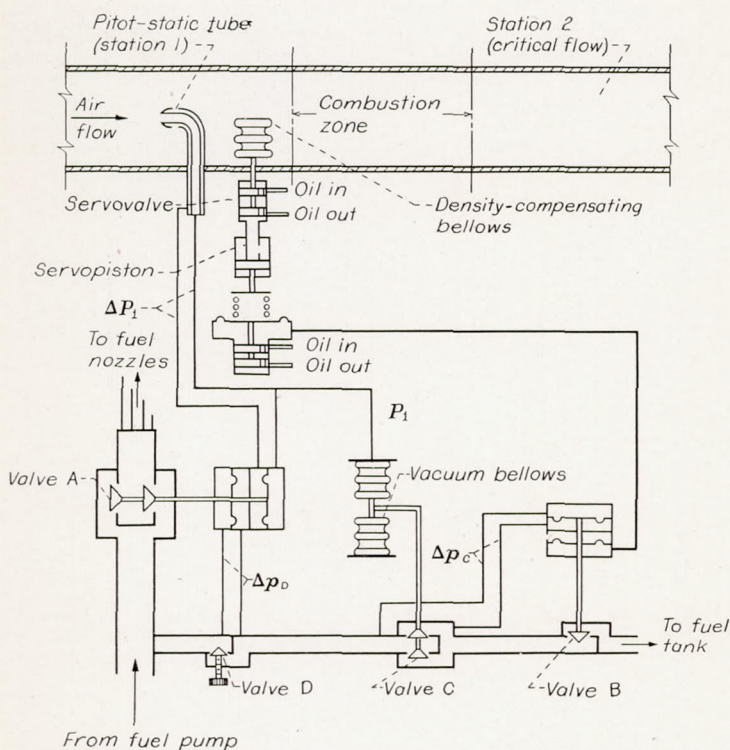


FIGURE 5.—Schematic diagram of temperature control based on critical-flow equation.

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CLEVELAND, OHIO, March 4, 1948.

APPENDIX A

SYMBOLS

The following symbols were used in the report:

A	area, sq ft
a	air or gas flow, lb/sec
B, G, K	constants
C	coefficient of discharge
E	area multiplier for thermal expansion of duct
f	fuel flow, lb/sec
g	acceleration due to gravity, ft/sec ²
M	Mach number
P	total pressure, lb/sq ft absolute
ΔP	total pressure minus static pressure, $P-p$
p	static pressure, lb/sq ft absolute
Δp	static-pressure drop across control valves
R	gas constant
r	ratio of static pressure to total pressure, p/P
T	total temperature, °R

t	static temperature, °R
V	velocity, ft/sec
γ	ratio of specific heats at constant pressure and constant volume
ρ	arbitrarily selected density, lb/cu ft
ρ_0	free-stream density, lb/cu ft
ρ_t	stagnation density, lb/cu ft
ϕ	conversion factor of hydraulic equation to compressible-flow equation
ϕ'	particular value of ϕ when $\rho = \frac{P}{RT}$

Subscripts:

A, B, C, D	valves A, B, C, and D in control systems
1	station 1 (before heating)
2	station 2 (after heating)

APPENDIX B

COMPRESSIBLE-FLOW EQUATIONS

The hydraulic equation for incompressible flow may be multiplied by an appropriate conversion factor ϕ to obtain the exact equation for compressible flow. This expression for the conversion factor ϕ may be derived from the compressible-flow equation by factoring out the hydraulic equation so that the remaining factor is the expression for the conversion factor ϕ .

Bernoulli's theorem for compressible flow may be written as

$$V = \left[\frac{2g\gamma}{\gamma-1} \left(\frac{P}{\rho_t} - \frac{p}{\rho_0} \right) \right]^{\frac{1}{2}} \quad (\text{B1})$$

The weight-flow rate is

$$a = A\rho_0 V \quad (\text{B2})$$

Substituting equation (B1) in equation (B2) and replacing stagnation density ρ_t with the equivalent adiabatic relation

$\rho_0 \left(\frac{P}{p} \right)^{\frac{1}{\gamma}}$ gives

$$a = A \left\{ 2g \left(\frac{\gamma}{\gamma-1} \right) \rho_0 \left[\frac{P}{\left(\frac{P}{p} \right)^{\frac{1}{\gamma}}} - p \right] \right\}^{\frac{1}{2}}$$

The free-stream density ρ_0 may be replaced by its equivalent

$\frac{p}{Rt}$ and the equation simplified:

$$\begin{aligned} a &= A \left\{ 2g \left(\frac{\gamma}{\gamma-1} \right) \frac{p}{Rt} \left[\frac{P^{\left(1-\frac{1}{\gamma}\right)} - p^{\left(1-\frac{1}{\gamma}\right)}}{p^{-\frac{1}{\gamma}}} \right] \right\}^{\frac{1}{2}} \\ a &= A \left\{ 2g \left(\frac{\gamma}{\gamma-1} \right) \frac{p^2}{Rt} \left[\frac{P^{\left(1-\frac{1}{\gamma}\right)} - p^{\left(1-\frac{1}{\gamma}\right)}}{p^{\left(1-\frac{1}{\gamma}\right)}} \right] \right\}^{\frac{1}{2}} \\ a &= A \left\{ 2g \left(\frac{\gamma}{\gamma-1} \right) \frac{p^2}{Rt} \left[\left(\frac{P}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right\}^{\frac{1}{2}} \end{aligned} \quad (\text{B3})$$

This expression for compressible flow may be written as

$$a = A\phi \sqrt{2g\rho(P-p)} \quad (\text{B4})$$

where

$$\phi = \left\{ \frac{1}{\rho(P-p)} \left(\frac{\gamma}{\gamma-1} \right) \frac{p^2}{Rt} \left[\left(\frac{P}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right\}^{\frac{1}{2}} \quad (\text{B5})$$

This expression of ϕ involves the density ρ , which also appears in equation (B4) and may be arbitrarily selected as a ratio involving total pressure P or static pressure p divided by total temperature T or static temperature t . For the case in which the density ρ is selected as p/RT , the conversion factor ϕ is designated as ϕ' and equation (B5) is simplified as follows:

$$(\phi')^2 = \frac{Tp\gamma \left[\left(\frac{P}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{t(P-p)(\gamma-1)} \quad (\text{B6})$$

The adiabatic relation of the temperatures is

$$\frac{T}{t} = \left(\frac{p}{P} \right)^{\frac{1-\gamma}{\gamma}} \quad (\text{B7})$$

This relation may be substituted into equation (B6) to obtain

$$(\phi')^2 = \frac{\left(\frac{p}{P} \right)^{\frac{1-\gamma}{\gamma}} \gamma \left[\left(\frac{P}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{\left(\frac{P-p}{p} \right) (\gamma-1)}$$

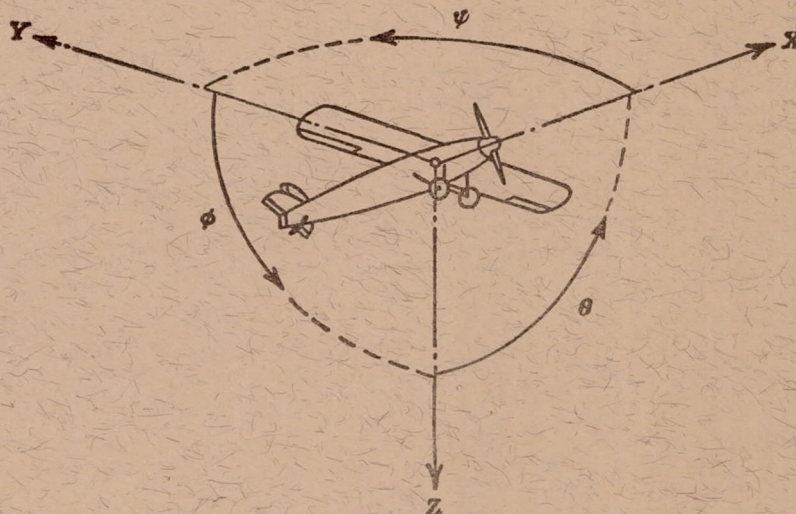
If the pressure ratio p/P is set equal to r this equation becomes

$$(\phi')^2 = \frac{\left(\frac{1-\gamma}{r^{\frac{1}{\gamma}}} \right) \left(\frac{1-\gamma}{r^{\frac{1}{\gamma}}} - 1 \right) \gamma}{\left(\frac{1}{r} - 1 \right) (\gamma-1)}$$

or

$$(\phi')^2 = \frac{r^{\frac{1}{\gamma}} \left(\frac{1-\gamma}{r^{\frac{1}{\gamma}}} - 1 \right) \gamma}{(1-r)(\gamma-1)} \quad (\text{B8})$$

A plot of the conversion factor ϕ' against the pressure ratio r is presented in figure 1, which shows the error that may be expected from neglecting ϕ in the hydraulic equation (B4) where the density ρ is p/RT . The greatest deviation of the conversion factor ϕ' from 1 occurs at the critical pressure ratio, at which ϕ' is approximately 0.945 for the ratio of specific heats γ equal to 1.3 for air at a temperature of 3000° R.



Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Sym- bol		Designation	Sym- bol	Positive direction	Designa- tion	Sym- bol	Linear (compo- nent along axis)	Angular
Longitudinal.....	X	X	Rolling.....	L	Y→Z	Roll.....	φ	u	p
Lateral.....	Y	Y	Pitching.....	M	Z→X	Pitch.....	θ	v	q
Normal.....	Z	Z	Yawing.....	N	X→Y	Yaw.....	ψ	w	r

Absolute coefficients of moment

$$C_l = \frac{L}{qbS}$$

(rolling)

$$C_m = \frac{M}{qcS}$$

(pitching)

$$C_n = \frac{N}{qbS}$$

(yawing)

Angle of set of control surface (relative to neutral position), δ . (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D Diameter

p Geometric pitch

p/D Pitch ratio

V' Inflow velocity

V_s Slipstream velocity

T Thrust, absolute coefficient $C_T = \frac{T}{\rho n^2 D^4}$

Q Torque, absolute coefficient $C_Q = \frac{Q}{\rho n^2 D^5}$

P Power, absolute coefficient $C_P = \frac{P}{\rho n^3 D^5}$

C_s Speed-power coefficient $= \sqrt[5]{\frac{\rho V_s^5}{P n^3}}$

η Efficiency

n Revolutions per second, rps

Φ Effective helix angle $= \tan^{-1} \left(\frac{V}{2\pi r n} \right)$

5. NUMERICAL RELATIONS

1 hp = 76.04 kg-m/s = 550 ft-lb/sec

1 metric horsepower = 0.9863 hp

1 mph = 0.4470 mps

1 mps = 2.2369 mph

1 lb = 0.4536 kg

1 kg = 2.2046 lb

1 mi = 1,609.35 m = 5,280 ft

1 m = 3.2808 ft